CHAIR FOR COMPILER CONSTRUCTION

Faster MPSoC Task Mapping via Symmetry Reduction

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Contents

1. Inspirations

Contents

1. Inspirations
2. Problem Statement

Contents

1. Inspirations
2. Problem Statement
3. Symmetry Reduction

Contents

1. Inspirations
2. Problem Statement
3. Symmetry Reduction
4. Representing and Extracting Symmetries

Contents

1. Inspirations
2. Problem Statement
3. Symmetry Reduction
4. Representing and Extracting Symmetries
5. The TMOR Problem

Contents

1. Inspirations
2. Problem Statement
3. Symmetry Reduction
4. Representing and Extracting Symmetries
5. The TMOR Problem
6. Experimental Results

Contents

1. Inspirations
2. Problem Statement
3. Symmetry Reduction
4. Representing and Extracting Symmetries
5. The TMOR Problem
6. Experimental Results
7. Further Research

Inspirations

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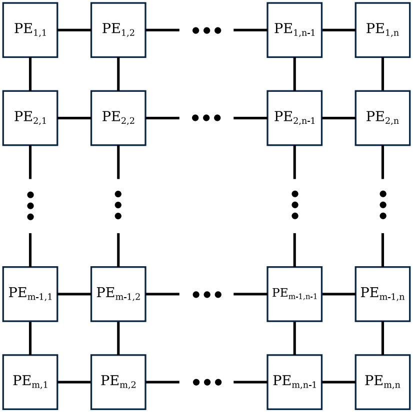
- Theoretical foundations: [Holt, 2005] and [East et al., 2019]
- Important optimizations: [Donaldson and Miller, 2009]

Problem Statement



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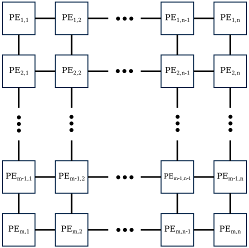

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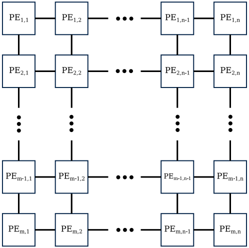

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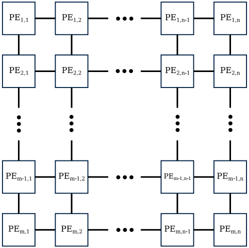

- Want to intelligently map tasks to processing elements
- Best choice depends on underlying optimality criteria
- Need to perform costly simulation!

Symmetry Reduction

- One approach:
  - Generate promising mappings based on previous simulations
  - $\rightarrow$ Traverse search space "intelligently"

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- Another approach:
  - Partition search space *by (partial) symmetry*
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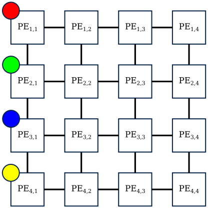
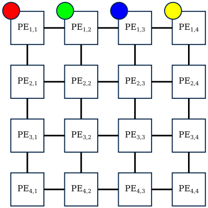
- Both approaches can be combined!

Symmetry Reduction



(a)



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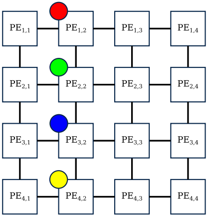
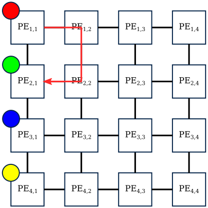


(c)

Symmetry Reduction



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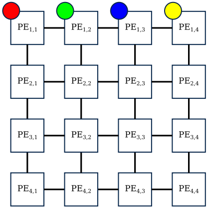

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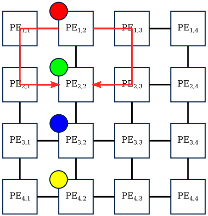
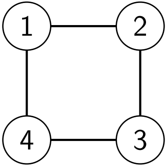


(c)

Representing Symmetries: Automorphism Groups



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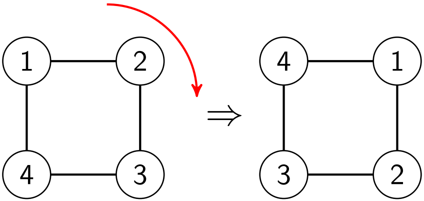
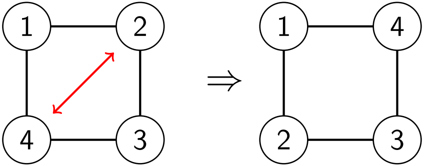
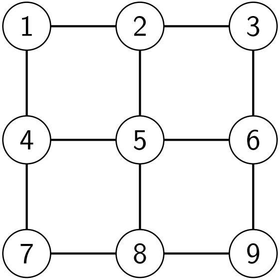


$\Rightarrow (1\ 2\ 3\ 4)$



$\Rightarrow (2\ 4)$

Representing Partial Symmetries: Partial Automorphism Inverse Monoids



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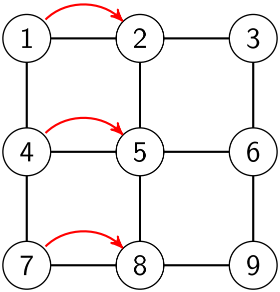

$\Rightarrow$ [1 2][4 5][7 8]

Extracting (Partial) Symmetries

- Describe MPSoC architecture as *architecture graph*

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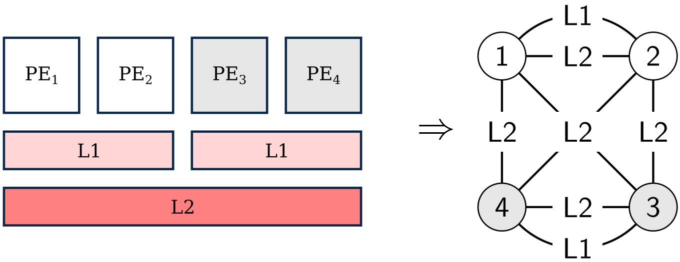
- Describe MPSoC architecture as *architecture graph*
- Determine Automorphism Group G:
  - Transform into “equivalent” vertex colored graph
  - Use *nauty* [McKay and Piperno, 2014]
- Determine Partial Automorphism Inverse Monoid M:
  - Construct search tree of possible generators
  - Prune certain subtrees
  - Not efficient enough in practice

Extracting (Partial) Symmetries



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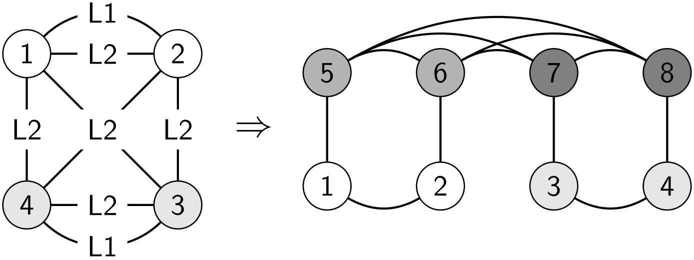

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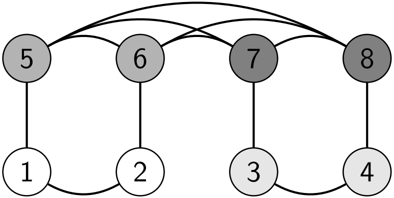

$$\Rightarrow \{(1\ 2)\cancel{(5\ 6)}, (3\ 4)\cancel{(7\ 8)}\}$$

Extracting (Partial) Symmetries

$$\{(1\ 2), (3\ 4)\} \Rightarrow \begin{array}{l} \text{Base: } [1, 3] \\ \text{Strong Generating Set: } \{(1\ 2), (3\ 4)\} \end{array}$$

The TMOR Problem I: Mapping Orbits

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- Orbits **partition** search space
- Reduce search space to set of *canonical orbit representatives*

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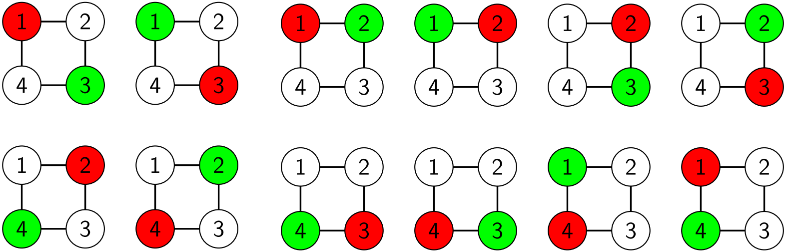
$$G = \langle\{(1\ 2\ 3\ 4), (2\ 4)\}\rangle$$



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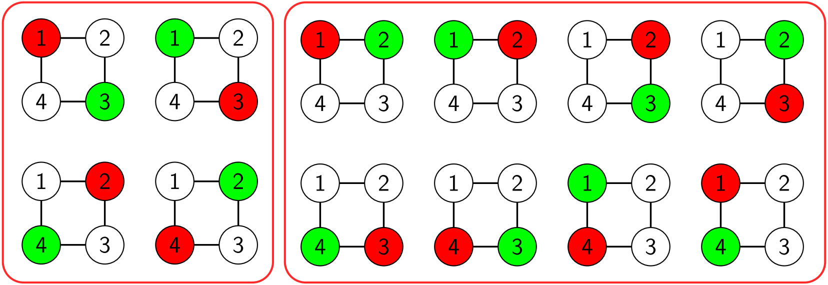

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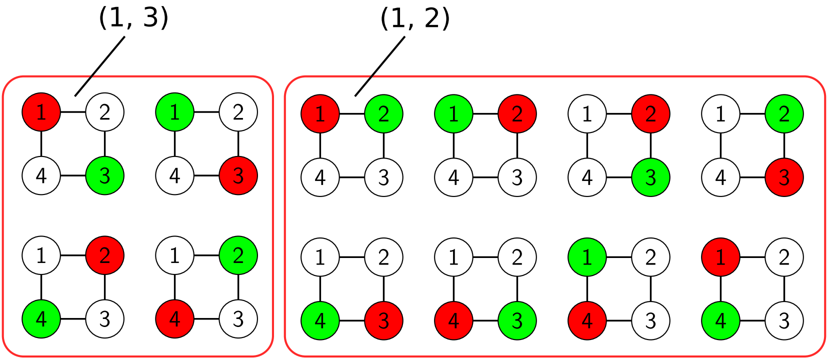

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procedure ORBIT_IDENTIFIER(t, reprs)

  repr $\leftarrow$ TMOR(t)

  if repr $\notin$ reprs **then**

    Append repr to reprs

  end if

  return index of repr in reprs

end procedure

The TMOR Problem II: Finding Canonical Orbit Representatives

- First approach: explicit orbit enumeration

```
procedure TMOR_ORBIT(t, G = ⟨S⟩)
    orbit ← {}

    while orbit is changing do
        for t′ ∈ orbit, g in S do
            orbit ← orbit ∪ {g(t′)}
        end for
    end while

    return min(orbit)
end procedure
```

The TMOR Problem II: Finding Canonical Orbit Representatives

- Second approach: group enumeration

```
procedure TMOR_ITERATE(t, G)
    repr ← t

    for g ∈ G do
        if g(t) < repr then
            repr ← g(t)
        end if
    end for

    return repr
end procedure
```

The TMOR Problem II: Finding Canonical Orbit Representatives

- Third approach: local search

```
procedure TMOR_LOCAL_SEARCH(t, G = ⟨S⟩)
    repr ← t

    while repr is changing do
        repr ← min({g(repr) | g ∈ S})
    end while

    return repr
end procedure
```

The TMOR Problem III: Optimization by Decomposition

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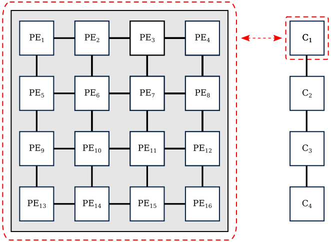

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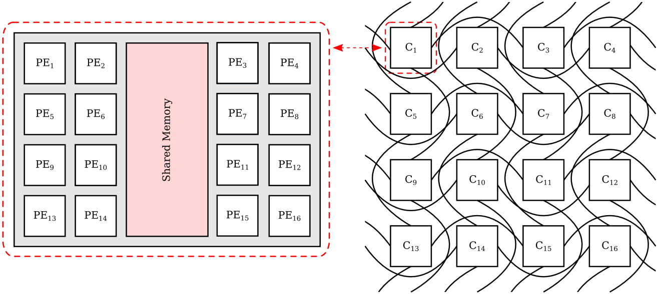

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  - Hierarchical architectures: *wreath products*
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- Idea based on [Donaldson and Miller, 2009]

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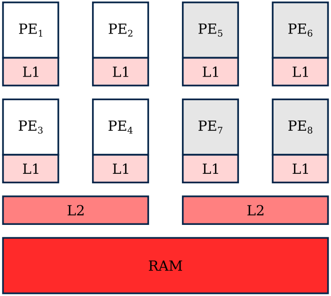

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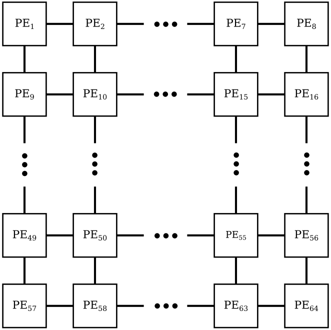

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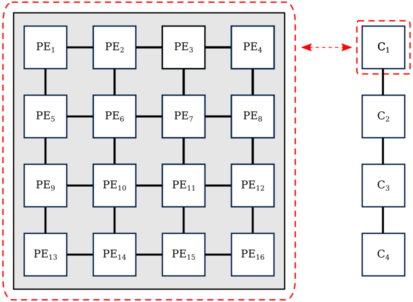

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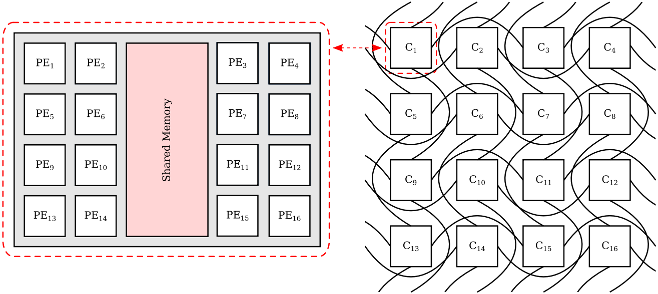
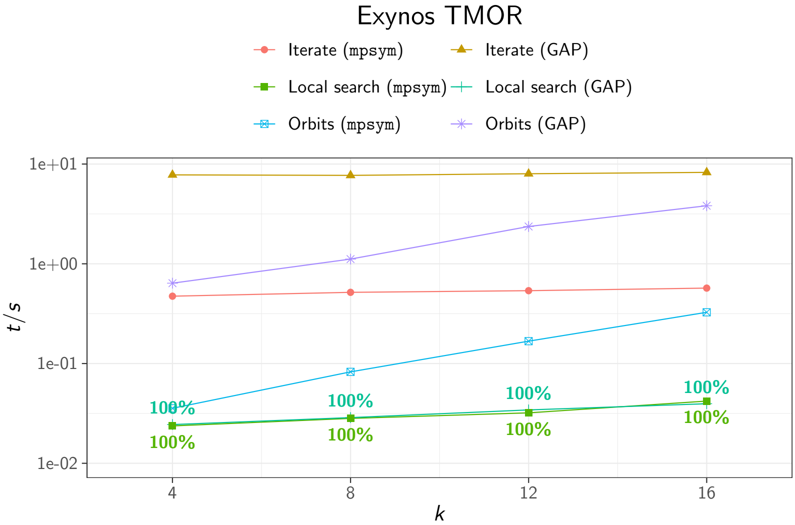


Experimental Results II: TMOR



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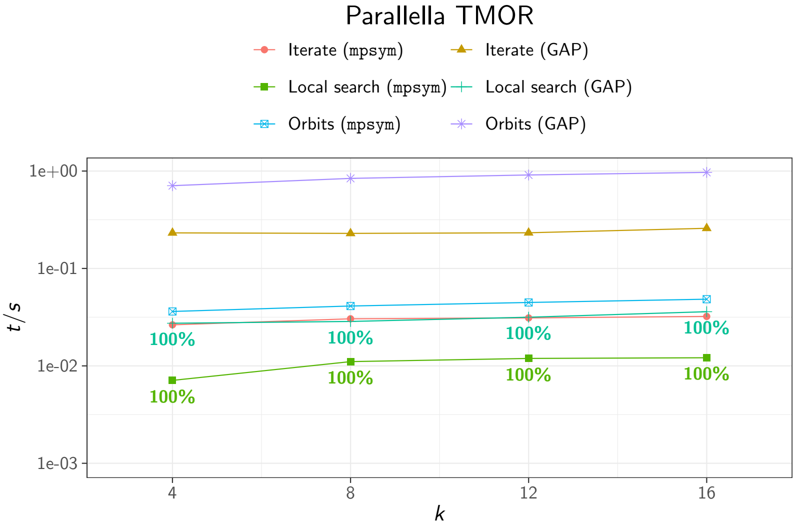

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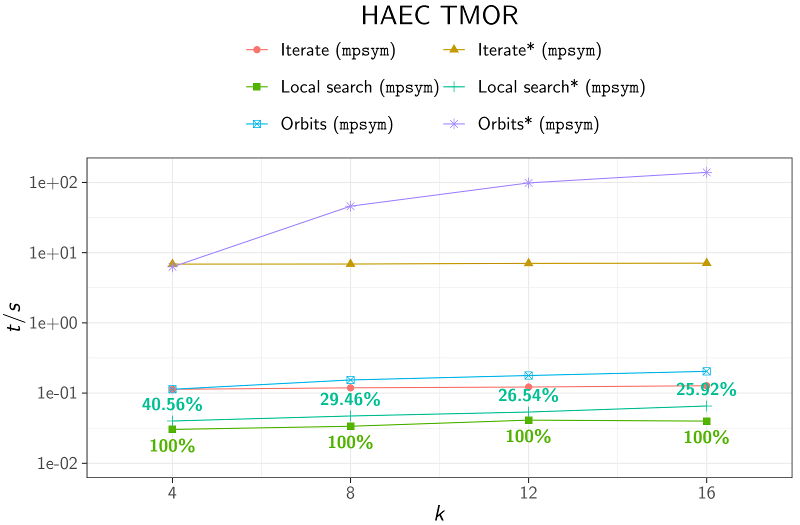

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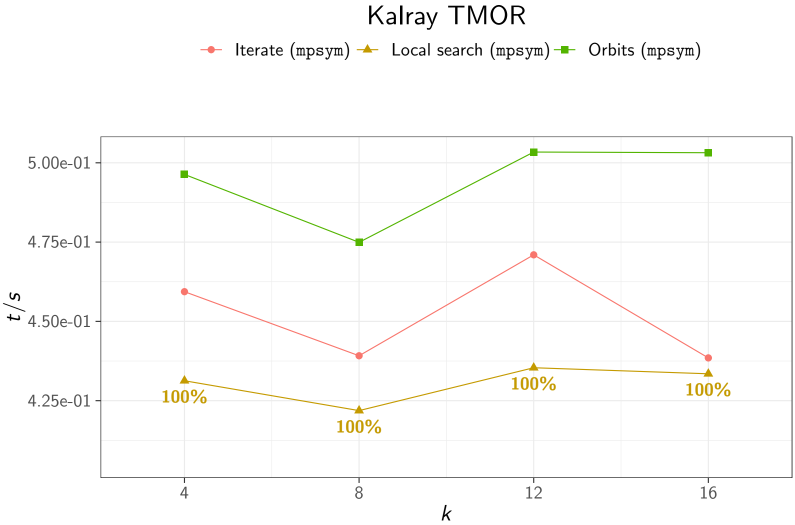

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  - `mpsym` outperforms GAP

Further Research

- Heuristic local search

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- Partial Automorphism discovery

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- Inverse Monoid enumeration

Further Research

- Heuristic local search
- Partial Automorphism discovery
- Inverse Monoid enumeration
- Interfacing GAP and C++

The End

Thank you for your attention!

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